Hertentamen 2 Statistiek voor KI/Inf/BMT (Külske)

Thursday 26-06-08, 14:00-17:00

All books, written notes, and all calculators allowed.
Cell phones and laptops not allowed.
Per exercise there are 9 points and 1 point for tree.

Good luck!

1. A fair coin is tossed. If a head turns up, a fair die is tossed; if a tail turns up, two fair dice are tossed. What is the probability that the face (or the sum of the faces) showing on the die (or the dice) is equal to six (9 points)?

2. a) When are two events A and B independent (2 points)?
   b) When are three events A, B, C independent (2 points)?
   c) Suppose you know that two events A and B are independent, $P(B) = 2P(A)$ and $P(A \cup B) = \frac{5}{8}$.
   Compute $P(A)$ and $P(B)$ (5 points)!

3. a) Give a definition in words of the Maximum Likelihood estimator! (2 points)

   We look at the 3 observations $1.0, 3.2, -0.9$ and we assume that they come from a double-sided exponential distribution with
   the density $f_\theta(y) = \frac{1}{2}e^{-|y-\theta|}$.

   b) Make a schematic drawing of the log-likelihood function (2 points)!
   c) What is the corresponding maximum likelihood estimate for θ (2 points)?
   d) Is it the same as the sample mean? If you take an even number of observations, will the maximum likelihood estimator be given by a single value? (3 points)

4. In 8 bottles of beer the following amount of beer (in litres) was found:
   $0.46, 0.48, 0.51, 0.48, 0.47, 0.49, 0.49, 0.46$.

   To test the null-hypothesi which asserts that
   "the beer bottles contain an average amount of 0.5 litres"

   the P-value of the sample shall be computed, assuming a normal sample with unknown variance.

   a) Should you perform a one-sided are a two-sided test? (2 points)
   b) What is the general definition of the P-value for a statistical test (2 points)
   c) What is the numerical value for the P-value in this case and what is your interpretation (5 points)?

5. Suppose that a random variable Y has the exponential pdf (density function) $f_Y(y) = e^{-y}$ for $y \geq 0$ and zero else.

a) What is the pdf of $\sqrt{Y}$ (3 points)?

b) Suppose that Y_1 and Y_2 are independent random variables which are both distributed as above. What is the variance of the random variable $V = Y_1 + Y_2$ (3 points).

c) Can you give the pdf of V (3 points)?